Ice Accretion Modeling Review

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Purpose of Presentation

- Give the audience a flavor of some of the engineering calculations involved in calculating how much ice can accrete on an aircraft wing for a given set of flight and weather conditions

When can aircraft icing occurs?

- **Icing** of an aircraft occurs when it flies through a cloud of small supercooled water droplets

- **Two types** of ice accretion mechanisms have been identified, resulting in two physically and geometrically different formations

Ice Accretion: Type I (Rime Ice)

- For low liquid water content, air temperature, and flight speed, the accreting ice is characterized by a **white opaque color** and a **low density** (less than 1 gm/cm^3).
- This formation is called **rime ice** and is more likely to occur on relatively streamlined shapes extending into the incoming air.
- **Rime ice** forms upon impact of the water droplets with the surface and is characterized by a freezing fraction of unity.

Ice Accretion: Type II (Glaze Ice)

- When both the liquid water content and the flight speed are high, while the air temperature is near freezing, the resulting ice formation will be characterized by a **clear color** and a **density near 1 gm/cm^3**.
- This mechanism of formation results in **glaze ice** which is usually associated with the presence of liquid water and a freezing fraction less than one.

Energy Fluxes on Aircraft Wings

- The energy added comprises terms which are due to freezing, aerodynamic heating, droplet kinetic energy, and external sources (such as the de-icing heater).
- The energy removed includes terms which are due to convection, evaporation, sublimation, droplet warming, and aft conduction.

Energy Fluxes on Aircraft Wings (Cont.)

- Both the wing leading edge and the after-body regions should be considered in any modeling effort.
- The figure illustrates the different modes of energy transfer to and from an accreting ice surface.

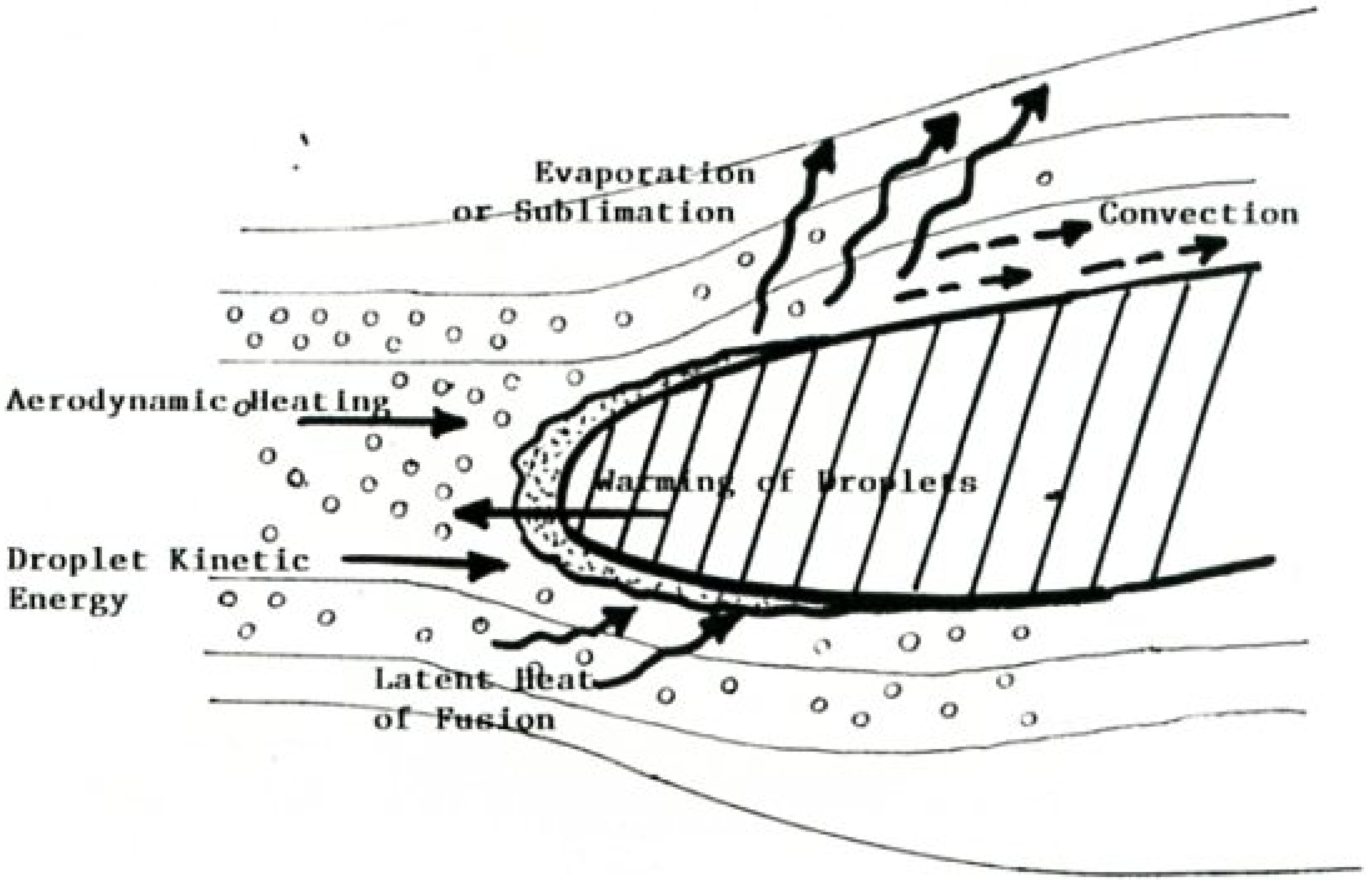

Figure 1. Modes of energy transfer to and from the leading edge of an airfoil

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Input Parameters Required for Modeling

- In order to be able to model icing on a wing, we will need knowledge of the following variables *a priori*:
  - altitude
  - flight speed V_∞ or the Mach number M_∞
  - volume median droplet diameter d_{drop}
  - equilibrium surface temperature t_s
  - wing configuration
  - angle of attack

Modeling Step No. 1
Compute Freestream Physical Properties

- For a given altitude, the freestream physical properties can be evaluated:
  - pressure p_∞
  - temperature t_∞
  - density ρ_∞
  - kinematic viscosity ν
  - thermal conductivity k

Modeling Step No. 2

Compute V_1/V_∞ vs. x/L

- Knowing the wing configuration and angle of attack we can determine the ratio of the boundary layer edge velocity to the freestream velocity, V_1/V_∞, as a function of the nondimensional chordwise distance, x/L (as described in Abbott el al. 1945).

Modeling Step No. 3
Compute p_1 and T_1

- Knowledge of V_1/V_∞ enables computing the pressure and temperature at the outer edge of the boundary layer:

$$\frac{p_1}{p_\infty} = \left[1 + \sqrt{\frac{\gamma - 1}{2}} M_\infty \left\{ \sqrt{\frac{\gamma - 1}{2}} M_\infty - \left(\frac{V_1}{V_\infty} \right) \right\} \right]^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_1}{T_\infty} = \left(\frac{p_1}{p_\infty} \right)^{\frac{\gamma - 1}{\gamma}}$$

Modeling Step No. 4

Compute p_1 in cases when C_p is given

- For some surfaces, the coefficient of pressure C_p along the surface may be available in lieu of the velocity ratio V_1/V_∞. In this case the pressure ratio should first be computed using the following expression:

$$\frac{p_1}{p_\infty} = 1 + \frac{\gamma}{2} M_\infty^2 C_P$$

Modeling Step No. 5

Compute V_1

- The velocity at the outer edge of the boundary layer should then be found using the equation:

$$\frac{V_1}{V_\infty} = \sqrt{\frac{2}{\gamma - 1}}\frac{1}{M_\infty}\left[1 + \left(\frac{\gamma - 1}{2}\right)M_\infty^2 - \left(\frac{p_1}{p_\infty}\right)^{\frac{\gamma-1}{\nu}}\right]$$

Modeling Step No. 6

Compute $K_{T,o}$

The modified inertia parameter, $K_{T,o}$, can be obtained from the following equation (Bowden et al. 1964):

$$K_{T,o} = 1.87x10^{-7}\left[\frac{1.15V_{\infty}}{\mu g}\right]^{0.6}\left[\frac{d_{drop}^{1.6}}{12\rho_{\infty}^{0.4}L}\right]$$

where the units are knots for V_{∞}, $lb_f.s/ft^2$ for μ, ft for the chord length, lb_m/ft^3 for ρ_{∞}, and ft/s^2 for g. The equation gives values within ±5% for droplet Reynolds numbers ranging from 25 to 1000.

Modeling Step No. 7

Compute the Local Collection Efficiency

- The local collection efficiency is computed employing the graphical relationships given in several references for a number of airfoils at different angles (see Brun et al. 1953 for the NACA 65A004 airfoil, and Brun et al. 1953 for the NACA 65-208 and 65-212 airfoils)

Local Collection Efficiency

- The local collection efficiency, β, is defined as the ratio between the locally impinging droplet flux and the freestream droplet flux.
- This efficiency is governed by the ratio of the inertia of the impinging droplets and their aerodynamic drag.
- It is primarily a function of the droplet size and distribution, water density and viscosity, freestream velocity, wing geometry, and angle of attack.

Modeling Step No. 8
Compute m''_i

- The local mass flux impinging on the surface may be computed from:

$$m_i'' = \beta W V_\infty$$

Modeling Step No. 9

Compute h_c

- The heat transfer coefficient, h_c, at the leading edge can be evaluated using a modified formulation which was originally introduced for a heated cylinder by Schmidt and Wenner (1943).
- This modified formulation replaces the cylinder diameter by the airfoil leading edge radius of curvature, d/2.

Equation for h_c

$$Nu_L = \mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.4}\left[1.14\left(\frac{L}{d}\right)^{0.5} - 2.353072\left(\frac{L}{d}\right)^{3.5}\left(\frac{s}{L}\right)^{3}\right]$$

Terms in the Heat Transfer Coefficient Equation at the Leading Edge

- Nu_L is the Nusselt number based on chord length L
- Re_L is the Reynolds number based on chord length L
- Pr is the Prandtl number
- s is the distance along the wing surface measured from the leading edge

Modeling Step No. 10
Compute h_c in the Aft Region (Laminar Regime)

- For the after region of the wing, two possibilities exist depending on the flow regime. For laminar flow, Martinelli et al. (1943) proposed the following equation:

$$Nu_L = 0.286\,\mathrm{Re}_L^{0.5}\left(\frac{V_1}{V_\infty}\right)^{0.5}\left(\frac{L}{s}\right)^{0.5}$$

Modeling Step No. 10
Compute h_c in the Aft Region (Turbulent Regime)

- For turbulent flow, the Nussett number expression in the after region may be written as:

$$Nu_L = 0.0296\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_L^{0.8}\left(\frac{V_1}{V_\infty}\right)^{0.8}\left(\frac{L}{s}\right)^{0.2}$$

Modeling Step No. 11
Compute the Air Thermal Conductivity and h_c

- Once the Nusselt number has been computed, the convective heat transfer coefficient may be determined for a given air thermal conductivity. Bowden et al. (1964) gave the following for the thermal conductivity:
  - T is in °R
  - k is in Btu/hr.ft.°F

$$k = \frac{0.001533\left(\frac{T}{1.8}\right)^{1.5}}{\frac{T}{1.8} + 245.4\left(10^{-12/\left(\frac{T}{1.8}\right)}\right)}$$

Modeling Step No. 12

Compute the Relative Heat Factor

- The relative heat factor, b, was originally introduced by Tribus (1949) and can be expressed as a nondimensional quantity of the impinging flux, the specific heat of liquid water, and the convective heat transfer coefficient:

$$b = \frac{m_i'' c_w}{h_c}$$

Modeling Step No. 13

Compute m''_f

- m''_f is the mass flux of the fraction of water impinging on the surface and freezing into ice and is given by:

$$m''_f = n_f m''_i$$

- where n_f is the freezing fraction

Modeling Step No. 14
Compute q_f

- The heat flux to the surface due to the freezing of the impinging water may be computed from:

$$q_f = m''_f \left[\lambda_f + c_i \left(t_{fz} - t_s \right) \right]$$

- where t_{fz} is the freezing temperature of water

Modeling Step No. 15
Compute “r”

- Compute the boundary layer recovery factor, r, as given by Hardy (1946):

$$r = \left[1 - \frac{V_1^2}{V_\infty^2} \left(1 - \mathrm{Pr}^{n_1} \right) \right]$$

- where n_1 is ½ for laminar boundary layers and 1/3 for turbulent boundary layers

Modeling Step No. 16

Compute q_{aero}

- The heat flux to the surface due to aerodynamic heating may be computed from:

$$q_{aero} = \frac{r h_c V_1^2}{2 g J c_P}$$

- where J is the mechanical equivalent of heat 778.26 ft.lb_f/Btu and c_P is the specific heat at constant pressure

Modeling Step No. 17
Compute q_{drop}

- Compute the heat flux to the surface due to droplet kinetic energy from:

$$q_{drop} = \frac{m_i'' V_\infty^2}{2gJ}$$

Modeling Step No. 18 Compute q_c

- The convective heat flux from the airfoil surface may be computed from:

$$q_c = h_c(t_s - t_1)$$

Some Observations

- Liquid water present on the wing surface may be a direct result of water impinging on the surface or it may be due to melting of some of the ice already in existence.
- Ice melting may occur due a variety of reasons such as aerodynamic heating, droplet kinetic energy, or the de-icing heater.

Some Observations (Cont.)

- In order to contain the complexity of this model, liquid water present on the surface will be assumed to result solely from direct impinging.
- Since the freezing fraction represents the portion of the impinging water freezing into ice, the remaining amount should represent the portion that remains as liquid.

Modeling Step No. 19
Compute the evaporation Potential

- The maximum amount of water that can be evaporated (or the evaporation potential) can be computed from the following equation according to Sogin (1954):

$$m''_{e,\max} = \frac{h_v p_1}{R_a T_f}\left[\frac{M_v}{M_a}\left\{\frac{p_{v,w}}{p_1 - p_{v,w}} - \frac{p_{v,\infty}}{p_\infty}\left(\frac{p_1}{p_1 - p_{v,w}}\right)\right\}\right]$$

More on the Evaporation Potential

- We assume that the water vapor behaves like a thermally perfect gas so that the thermodynamic properties of the vapor can be calculated as though the air were not present.
- We allow the densities of the water vapor at the surface and the boundary layer edge to be evaluated in terms of the partial pressures of the vapor
- We account for the influence of induced convection

Modeling Step No. 20

- Compute the water vapor pressure using empirical correlations. For the temperature range 492 ≤T≤ 672°R, Pelton and Willbanks (1972) provided the following equation:

$$p_{v,w} = 2117\left(\frac{672}{T}\right)^{5.19} \exp\left[-9.06\left(\frac{\lambda_e}{T} - 1.4525\right)\right]$$

- where T is in °R, $p_{v,w}$ is in lb_f/ft^2 absolute

Modeling Step No. 21

- Compute the latent heat of vaporization from:

$$\lambda_e = 1352.3 - 0.5696T + 0.0839x10^{-3}T^2 + 0.0927x10^{-7}T^3$$

Modeling Step No. 22

- For a supercooled liquid at a temperature less than 492°R, Dorsey (1940) provided this correlation:

$$p_{v,w} = 2117\exp\left[2.3\left\{A_1 + \frac{A_2}{\left(\frac{T}{1.8}\right)} + \frac{A_3\left(\left(\frac{T}{1.8}\right)^2 - A_7\right)}{\left(\frac{T}{1.8}\right)}\left(10^{\left[A_4\left(\left(\frac{T}{1.8}\right)^2 - A_7\right)^2\right]} - 1\right) + A_5\left(10^{\left[A_6\left(374.11 - \left(\frac{T}{1.8}\right)\right)^{5/4}\right]}\right)\right\}\right]$$

- where $A_1 = 5.4266514$, $A_2 = -2005.1$, $A_3 = 1.3869 \times 10^{-4}$, $A_4 = 1.1965 \times 10^{-11}$, $A_5 = -4.4 \times 10^{-3}$, $A_6 = -5.7148 \times 10^{-3}$, and $A_7 = 2.937 \times 10^5$.

Modeling Step No. 23

- Compute the mass transfer coefficient, h_v, which may be related to the heat transfer coefficient, h_c, employing the Lewis analogy:

$$h_v = \frac{h_c}{\rho c_P Le^{2/3}} = \frac{h_c}{\rho c_P \left(\frac{\alpha}{D}\right)^{2/3}}$$

Modeling Step No. 24

- Compute the coefficient of mass diffusion of water vapor in air, D, using the following empirical relationship (ASHRAE 2009):

$$D = \frac{0.00215}{p}\left(\frac{T^{2.5}}{T+441}\right)$$

- where the pressure is in psia, the temperature is in °R, and the diffusion coefficient is in ft^2/hr.

Modeling Step No. 25

- Compute the maximum amount of ice that can be sublimated (or the sublimation potential) from:

$$m''_{s,\max} = \frac{h_v p_1}{R_a T_f}\left[\frac{M_v}{M_a}\left\{\frac{p_{v,i}}{p_1 - p_{v,i}} - \left(\frac{p_{v,\infty}}{p_\infty}\right)\frac{p_1}{p_1 - p_{v,i}}\right\}\right]$$

- Similar calculations are performed for sublimation. Many more calculation steps are needed.

Conclusions

- We have presented a summary of some engineering calculations used in modeling icing on aircraft wings
- More sophisticated models require more intense mathematical analysis
- Time-dependent analysis can be modeled using quasi-steady state calculations under certain conditions